

Charmed baryon in a string model

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Charm spectroscopy has studied under a string model. Charmed baryons are composed of diquark and charm quark which are connected by a constant tension. In a diquark picture, the quantum numbers J^P of confirmed baryons are well assigned. We give energy predictions for the first and second orbital excitations. We see some correspondences with the experimental data. Meanwhile, we have obtained diquark masses in the background of charm quark which satisfy a splitting relation based on spin-spin interaction.

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I. INTRODUCTION

Charm spectroscopy has revived since 2000. Many new excited charmed baryon states have been discovered by CLEO, BaBar, Belle and Fermilab. Masses of ground states as well as many of their excitations are known experimentally with rather good precision. However charmed baryons have narrow widths and non of their spin or parity are measured except the $\Lambda_c(2880)$ [1]. The assignments listed in the PDG book almost are based on quark model[2]. Theoretically, the study of heavy baryons has a long story[3, 4, 5]. Heavy baryons provide a laboratory to study the dynamics of the light quarks in the environment of heavy quark, such as their chiral symmetry[6]. The studies of heavy baryons also help us to understand the nonperturbative QCD[7]. Furthermore, it really is an ideal place for studying the dynamics of diquark.

The concept of diquark appeared soon after the original papers on quarks[8, 9, 10]. It was used to calculate the hadron properties. In heavy quark effective theory, two light quarks often refer to as diquark, which is treated as particle in parallel with quark itself. There are several phenomenal manifestations of diquark: the $\Sigma - \Lambda$ mass difference, the isospin $\Delta I = 1/2$ rule, the structure function ratio of neutron to proton, *et al.* [11, 12, 13, 14]. In this diquark picture, charmed baryons are composed of one diquark and one charm quark.

Selem and Wilczek[13, 14] have generalized the famous Chew-Frautschi formula by considering diquark and quark connected by a relativistic string with constant tension T and rotating with angular momentum L . The string is responsible for the color confinement and is also called as

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loaded flux tube if the two ends get masses. In the limit of zero diquark and quark masses, the usual Chew-Frautschi relationship $E^2 \sim L$ appears. They have investigated the $N - \Delta$ spectrum and concluded that “large L spectroscopy would give convincing evidence for energetically significant diquark correlations” [14]. In hadrons containing one heavy quark, diquark ought to be a better approximation than in hadrons containing only light quarks. However Selem and Wilczek have given only a short discussion on the Λ_c spectrum. In this paper, we accept the diquark concept and use their relativistic string model to study the charmed baryon spectroscopy.

In the following, we introduce firstly the diquarks and the string model in section II. We give our analysis of the doublets and give the quantum number assignments based on diquark assumption. Numerical results are showed in section III. In the end summary and discussion are given.

II. DIQUARK AND STRING MODEL

A. Diquarks and mass splitting

The single-charmed baryons composed of one diquark and one charm quark. In literatures, there are two kinds of diquarks: the good diquark with spin zero and the bad diquark with spin one. The good diquark is more favorable energetically than the bad one, which is indicated by both the one-gluon exchange and instanton calculations. In $SU(3)_f$, the good diquark has flavor-spin symmetry $\bar{\mathbf{3}}_F \bar{\mathbf{3}}_S$ while the bad diquark $\mathbf{6}_F \mathbf{6}_S$. To give a color singlet state, both kinds of diquark have the same color symmetry $\bar{\mathbf{3}}_C$. In the following we use the $[qq']$ to denote a good diquark, while (qq') the bad and l to denote either u quark or d quark.

In this diquark picture, Λ_c has a $[ud]$ component, while Σ_c , (ll') and Ω_c , (ss) . For Ξ_c , either kind of diquarks, good or bad, can be formed. Heavy baryons always have been obtained by continuum production[7]. So, the lowest baryons discovered are more likely to be ground states. These are $\Lambda_c(2285)$ and $\Xi_c(2470)$. Three doublets $\Sigma_c(2455, 2520)$, $\Xi_c(2578, 2645)$ and $\Omega_c(2768, 2698)$ would also be states with $L = 0$, if they are composed of bad diquark and charm quark. And we assign the doublets $\Lambda_c(2595, 2628)$ and $\Xi_c(2790, 2815)$ with $L = 1$.

The good diquark with spin zero has no spin interaction with the charm quark. So, the lowest energy is a singlet. Only the $L - S$ coupling may make the energy split[12]:

$$\mathcal{H}(q_c, [qq']) = \mathcal{K}_{[qq']} 2\vec{L} \cdot \vec{S}_c, \quad (1)$$

where the coefficient $\mathcal{K}_{L,(qq')}$ depends on the diquark and charm quark masses. This interaction splits baryon with orbital angular momentum L to baryons with $J = L + 1/2$ and $L - 1/2$. And the parity is $P = (-1)^L$. For the bad diquark, the spin-spin interaction is:

$$\mathcal{H}(q_c, (qq')) = \mathcal{G}_{(qq')} 2\vec{S}_{(qq')} \cdot \vec{S}_c, \quad (2)$$

where $\vec{S}_{(qq')}$ is the spin of the bad diquark, and the coefficient $\mathcal{G}_{(qq')}$ depends on the diquark and charm quark masses. This spin-spin interaction also lead to a doublet in the spectrum. Since in our assignments, there is no $L > 0$ multiplet of bad diquark and for simplicity, we will not discuss the splitting caused by $L - S$ coupling for baryons containing bad diquark.

We have relation $< 2\vec{j}_1 \cdot \vec{j}_2 > = J(J+1) - j_1(j_1+1) - j_2(j_2+1)$, with $\vec{J} = \vec{j}_1 + \vec{j}_2$. It is easy to deduce the mass difference of a doublet. For example, when $j_1 = 1$, $j_2 = 1/2$, they are $M_0 + \Delta$ and $M_0 - 2\Delta$, with Δ being \mathcal{G} or \mathcal{K} . Taking a doublet as input, we can obtain the Δ and M_0 . And it is not the masses of the doublet, but this M_0 which enter into the string model.

B. The string model

In 1960 Chew and Frautschi conjectured that the strongly interacting particles fall into families where the Regge trajectory functions were straight lines: $E^2 = \sigma + kL$ with the same constant k for all the trajectories. The straight-line Regge trajectories with σ zero were later understood as arising from massless endpoints on rotating relativistic strings at speed of light transversely. A non-zero values of σ may include zero-point energy for string vibrations and loaded endpoints.

In Selem and Wilczek's model[13, 14], the two ends of the string have masses m_1 and m_2 respectively, with constant string tension T . The rotating angular momentum is L with angular velocity ω . If the diquark and charm quark are away from the center of rotation at distances R_1 and R_2 , the energy of the system is:

$$E = \sum_{i=1,2} (m_i \gamma_i + \frac{T}{\omega} \int_0^{\omega R_i} \frac{1}{\sqrt{1-u^2}} du), \quad (3)$$

where γ_i is the usual Lorentz factor:

$$\gamma_i = \frac{1}{\sqrt{1 - (\omega R_i)^2}}. \quad (4)$$

The angular momentum can be written as:

$$L = \sum_{i=1,2} (m_i \omega R_i^2 \gamma_i + \frac{T}{\omega^2} \int_0^{\omega R_i} \frac{u^2}{\sqrt{1-u^2}} du). \quad (5)$$

Furthermore, we have formula relating the tension and the angular velocity:

$$m_i \omega^2 R_i = \frac{T}{\gamma_i^2}. \quad (6)$$

From equation (6), we see that ωR_i can be replaced by m_i , γ_i and T/ω . Solving the equations (4) and (6), we can express the γ_i by T/ω and m_i :

$$\gamma_i = \sqrt{\frac{1}{2} + \frac{\sqrt{1 + 4(T/m_i\omega)^2}}{2}} \quad (7)$$

Now, we have two equations, (3) and (5), and 6 parameters, E , L , m_1 , m_2 , T and ω . These equations are more useful than the Chew-Frautschi formula for they make us able to extract the diquark masses.

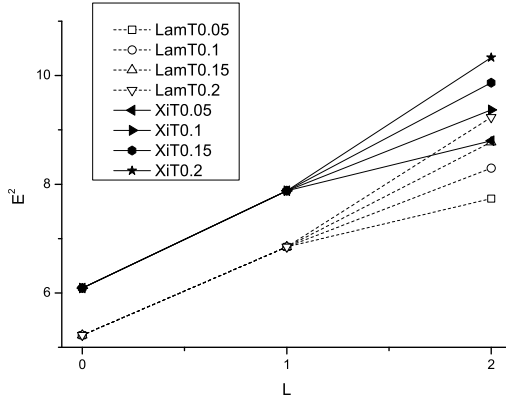


FIG. 1: Plot of $E^2 \sim L$ for Λ_c and Ξ_c with $m_c = 1.7 \text{ GeV}$ and $T = 0.05 \sim 0.20$.

For very light mass, it appears that $\omega \rightarrow \infty$ as $L \rightarrow 0$ and the Chew-Frautschi relationship is recovered as $E^2 = (2\pi T)L$. For other cases, such as the first corrections at small masses see Ref.[13, 14]. A reduced formula has being used to study the charmed meson spectroscopy[15]. In this paper we will solve the equations numerically.

III. NUMERICAL RESULTS

A. Λ_c and Ξ_c with good diquark

We chose $L = 1$ doublet as input to solve the equations (3) and (5). Firstly, we take the m_c and T as free parameters to get the diquark mass. The string tension T is universal for baryons with the same components. Then we use these three parameters to give energy predictions for $L = 2$. And we find that the T is almost equal for Λ_c and Ξ_c if we choose the value which give a linear Regge trajectory $E^2 \sim L$ or linear $(E - M)^2 \sim L$. The last relation was given by Selem and Wilczek, which can be obtained by expanding the right hands of equations (3) and (5) in $m\omega/T$ for terms of light diquark and in $T/(m\omega)$ for terms of charm quark. The results are listed in Table I and II. The two kinds trajectories are both linear since the energies for $L = 0, 1, 2$ at $T = 0.1$ form a arithmetic progression and with small common difference. We plot the two kinds trajectories with $M_c = 1.7 \text{ GeV}$ for example, on Figure 1 and 2.

The energies $M_0^{L=1}$ is 2.617 GeV for doublet $\Lambda_c(2595, 2628)$ with $\mathcal{K}_{[u]} = 11 \text{ MeV}$. The numerical result for $L = 2$ are $M_0 = 2.879 \text{ GeV}$ which gives doublets $\Lambda'_c(2846, 2901)$ with the splitting mass formulas $M_0 + 2\mathcal{K}$ and $M_0 - 3\mathcal{K}$. Here, we use a prime to indicate our theoretic prediction. We think the $\Lambda_c(2880)$ and $\Lambda_c(2940)$ to be a doublet with $L = 2$, since the splitting gives $\mathcal{K}_{[u]} = 12 \text{ MeV}$ which is near equal to the result from mass difference of $\Lambda_c(L = 1)$. Then their mass difference

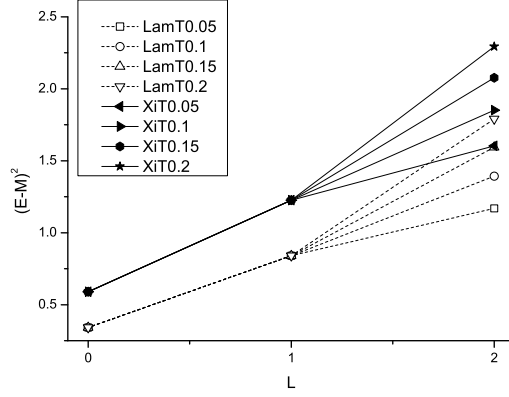


FIG. 2: Plot of $(E - M)^2 \sim L$ for Λ_c and Ξ_c with $m_c = 1.7 \text{ GeV}$ and $T = 0.05 \sim 0.20$.

$\Lambda_c: m_{[u]}, M_0^{L=2}$	T=0.05	0.1
Mc=1.5	0.873, 2.781	0.737, 2.878
1.6	0.770, 2.781	0.630, 2.878
1.7	0.665, 2.781	0.518, 2.879
1.8	0.559, 2.782	0.403, 2.881

$\Lambda_c: m_{[u]}, M_0^{L=2}$	T= 0.15	0.2
Mc=1.5	0.624, 2.959	0.520, 3.032
1.6	0.509, 2.960	0.395, 3.034
1.7	0.387, 2.962	0.255, 3.038
1.8	0.253, 2.967	0.080, 3.043

TABLE I: Good diquark mass of $[u]$ and energy for $\Lambda_c(L = 2)$ neglecting the spin-spin interaction. Mass unit is in GeV.

gives $M_0^{L=2} = 2917 \text{ MeV}$ which is 38 MeV large than our prediction. However, it can be wiped out given $T = 1.2$, see Table III.

The linear fit is:

$$\Lambda_c : E^2 = 1.632L + 5.220, \quad (8)$$

with χ/Dof almost being zero. So we predict that the quantum numbers J^P of doublet $\Lambda_c(2882, 2940)$ are $3/2^+$ and $5/2^+$.

For Ξ_c , $M_0^{L=1}$ is 2.807 GeV with $\mathcal{K}_{[ls]} = 8.3 \text{ MeV}$. If $T = 0.12$, we get $M_0^{L=2} = 3.094 \text{ GeV}$ and the doublet is $\Xi'_c(3069, 3111)$. And the Regge trajectory is

$$\Xi_c : E^2 = 1.736L + 6.115. \quad (9)$$

$\Xi_c: m_{[ls]}, M_0^{L=2}$	T=0.05	0.1
Mc=1.5	1.070, 2.968	0.940, 3.063
1.6	0.969, 2.967	0.835, 3.062
1.7	0.866, 2.966	0.730, 3.061
1.8	0.762, 2.967	0.620, 3.063

$\Xi_c: m_{[ls]}, M_0^{L=2}$	T=0.15	0.2
Mc=1.5	0.835, 3.142	0.738, 3.214
1.6	0.726, 3.141	0.625, 3.213
1.7	0.614, 3.141	0.505, 3.214
1.8	0.498, 3.143	0.380, 3.216

TABLE II: Good diquark mass of $[ls]$ and energy for $\Xi_c(L=2)$ neglecting the spin-spin interaction. Mass unit is in GeV.

$B_{(qq')}$	Λ_c	Ξ_c
\mathcal{K}/MeV	11	8.3
$m_{[qq']} / \text{GeV}$	0.465	0.682
$M_0^{L=1} / \text{GeV}$	2.617	2.807
$M_0^{L=2} / \text{GeV}$	2.913	3.094
mass splitting	L=1: $M_0 + \mathcal{K}$ and $M_0 - 2\mathcal{K}$	
	L=2: $M_0 + 2\mathcal{K}$ and $M_0 - 3\mathcal{K}$	

TABLE III: Good diquark masses and predictions for masses at $L=2$ with $T=0.12$ and $m_c=1.7 \text{ GeV}$. By using the mass splitting formula at $L=1$, $M_c^{L=1}$ and \mathcal{K} are easy to be derived.

The nearest experimental data are $\Xi_c(3080)$ and $\Xi_c(3123)$ only about 10 MeV larger than our predictions. So, we take $\Xi_c(3080, 3123)$ as a doublet with $J^P = 3/2^+$ and $5/2^+$.

B. Σ_c , Ξ_c and Ω_c with bad diquark

For Σ_c , Ω_c and Ξ_c with bad diquark, we have to take the $L=0$ doublets as input for the lack of data. When $L \rightarrow 0$, we have $\omega \rightarrow 0$, $R \rightarrow 0$ and $E \rightarrow m_1 + m_2$ from which we can deduce the bad diquark masses. We see from Table I and Table II that energies are more depending on T not on quark masses. So we take charm quark mass to be 1.7 GeV . The numerical results with $T=0.12$ are listed in Table IV. Linear fits of the three groups of baryon masses are:

$$\Sigma_c : E^2 = 1.987L + 6.326,$$

$$\Xi_c : E^2 = 2.035L + 6.967,$$

$B_{(qq')}$	Σ_c	Ξ_c	Ω_c
\mathcal{G}/MeV	21.7	22.3	23.3
$m_{(qq')} / \text{GeV}$	0.798	0.923	1.045
M_0 / GeV	2.498	2.623	2.745
$M_0^{L=1} / \text{GeV}$	2.913	3.029	3.144
$M_0^{L=2} / \text{GeV}$	3.196	3.309	3.421

TABLE IV: Results of the string model using the ground states as input and with $T = 0.12$.

$B_{(qq')}$	Σ_c	Ξ_c	Ω_c
\mathcal{G}/MeV	21.7	22.3	23.3
$m_{(qq')} / \text{GeV}$	0.739	0.858	0.975
M_0 / GeV	2.498	2.623	2.745
$M_0^{L=1} / \text{GeV}$	2.815	2.926	3.036
$M_0^{L=2} / \text{GeV}$	3.100	3.201	3.302

TABLE V: Results of using Regge trajectory with slope being 1.684. And the diquark masses are derived by taking the $M_0^{L=1}$ as input.

$$\Omega_c : E^2 = 2.084L + 7.624,$$

with χ/Dof being about 0.04 for each fit. The slopes are almost equal but a little larger than 1.632 and 1.736, the slopes for fitting the spectra of Λ_c and Ξ_c containing good diquark. However, the diquark masses are so heavy. And it is unreasonable for a string with zero length. So, when $L \rightarrow 0$ and $R \rightarrow 0$, the string model would not be a good approximation.

In the end, we give the mass predictions for these baryons using linear Regge trajectory, though there are arguments that hadronic Regge trajectories are nonlinear[16]. We take the slope to be the average of the slopes for good diquark baryons, that is 1.684. Then use equations (3) and (5) with $L=2$ to extract the diquark masses. Results are showed in Table V. In PDG book, there is $\Sigma_c(2800)$ with question mark which is a little lower than our prediction for $\Sigma_c(2815, L=1)$. And note that we have neglected here all the angular momentum interactions.

C. Diquark masses

The good and bad diquark masses are listed in Table III and Table V. Bad diquarks are heavier than good diquarks and diquark with heavier flavor quark is heavier than the light one. These diquark masses are sensitive to the background, i.e. the charm quark mass. However, they still satisfy the relation $(ud) - [ud] > (us) - [us]$ which was expected from spin-spin interaction that the mass difference would be strongest for lightest quarks[13, 14].

We can adopt the string model to the charmed mesons. The non-strange mesons $D(2400, 2420, 2430, 2460)$ with positive parity would be a multiplet of $L = 1$. The meson $D(2460, J^P = 2^+)$ thus has total spin $S = 1$ and $\langle 2L \cdot S \rangle = 0$. The same is for charmed and strange meson $D_s(2573)$. Using this two states as input, we have derived the quark masses which are 0.332 GeV for up and down quark and 0.468 GeV for strange quark. The diquark masses can be defined by $M_D = M_{q1} + M_{q2} + E_{12}$, with E_{12} being the binding energy. We see that the good diquarks have negative binding energies while the bad positive. This is consistent with result that comes from spin-dependent colormagnetic interactions of two quarks. The interactions are attractive in a spin-0 state and repulsive in a spin-1 state[17].

IV. SUMMARY AND DISCUSSION

We have used a diquark picture and a string model to study the charmed baryon spectroscopy. The many doublets in the spectroscopy are the results of S-S or L-S interactions. With string tension $T = 0.12$ we have given predictions for the good diquark baryons with $L=2$ which have some experimental correspondences. The possible state $\Sigma_c(2800)$ would be the first orbital excitation of Σ_c . The quantum number J^P assignments for $L=0$ and $L=1$ baryons from a diquark picture are the same as PDG book. By using the string model, we have extracted the diquark masses which satisfy the expected relation $(ud) - [ud] > (us) - [us]$.

However, there is one problem. Our prediction for $\Lambda_c(2880) J^P = 3/2^+$ is contradicted with the experimental result and Selem's assignment with $J^P = 5/2^+[1, 14]$. If it is confirmed by later experiments, we must reconsider our diquark picture or mass splitting formula based on angular momentum interactions.

V. ACKNOWLEDGMENTS

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